Transport and condensation of driven tracers in a narrow channel

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Asaf Miron, DM , H. A. Posch, JSTAT 063216 (2020)
Asaf Miron, DM, J. Phys. A 54, 025001 (2021)
Asaf Miron, DM, H. A. Posch, PRE 104, 024123 (2021)

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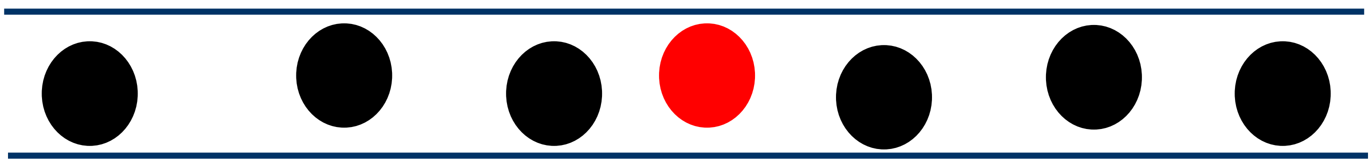
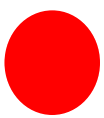
KIAS , July 25-28, 2022

Single file dynamics

Particles in a narrow channel
Diffusing symmetrically
No overtaking

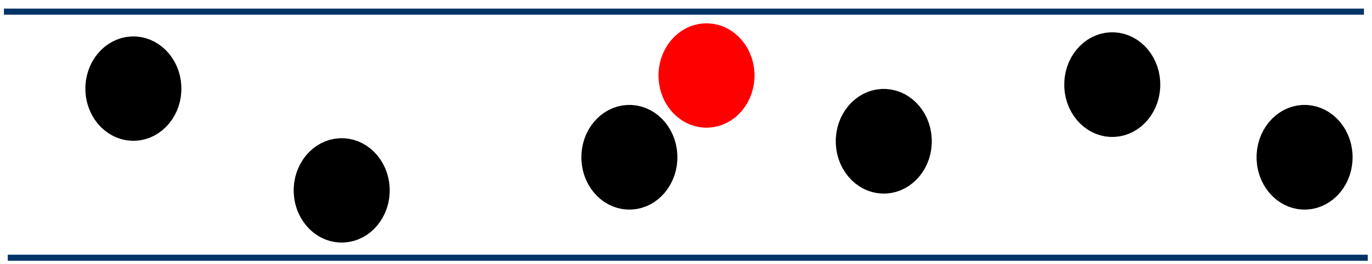




Tagged particle

A tagged particle is sub-diffusing $\langle x^2 \rangle \propto \sqrt{t}$

for wide enough channel which allows overtaking



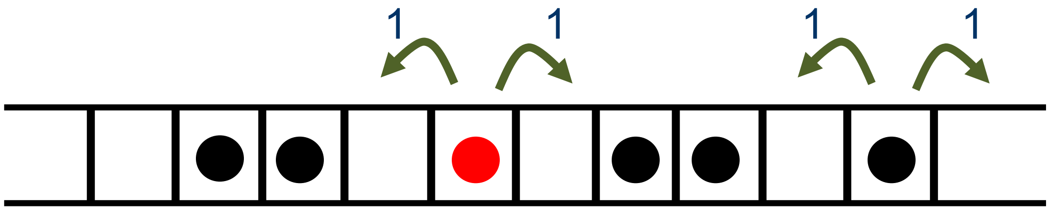
diffusion is restored

$$\langle x^2 \rangle \propto t$$

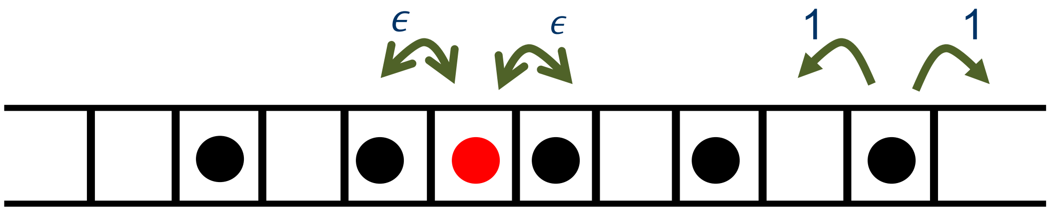
no matter how small is the overtaking rate

Lattice model of single file dynamics (SSEP)

Single file:



With overtaking (exchange):



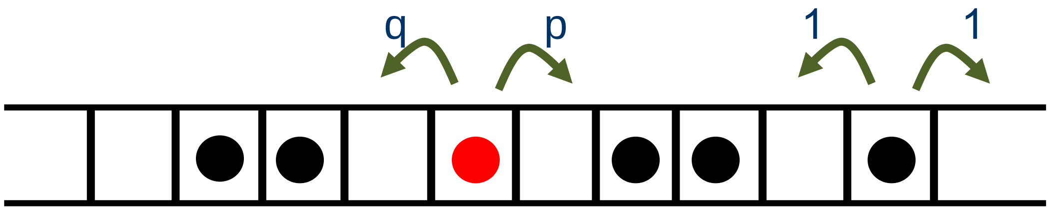
$$\langle x^2 \rangle = D(t)t$$

At large t

$\epsilon = 0$: $D(t) \sim 1/\sqrt{t}$

$\epsilon \neq 0$: $D(t) = const.$

Driven Tracer – no overtaking



The dynamics of the tracer is that of single file:

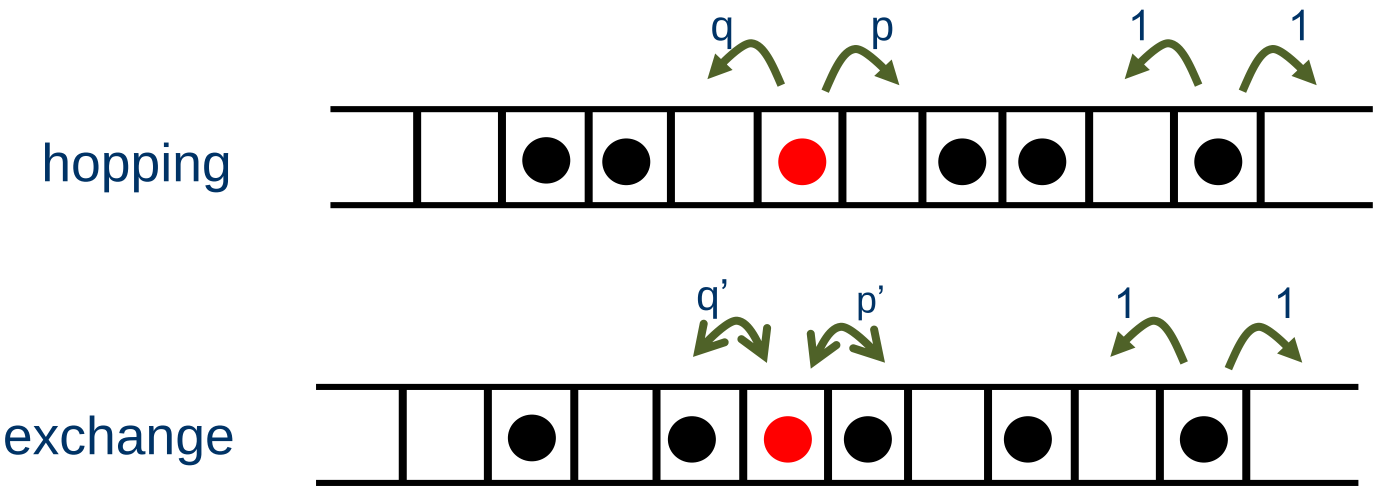
line with $L = \infty$: $\langle \Delta x^2 \rangle \propto \sqrt{t}$

$\langle x \rangle \propto \sqrt{t}$ (zero velocity v)

ring with $L < \infty$: $v \propto 1/L$ finite velocity

Burlatsky, Oshanin et al (1992, 1997); Landim, Olla Volchan (1998); Benichou, et al (2013);
Cividini, Kundu, Majumdar DM (2016);

Driven Tracer with overtake (particle exchange)



How does the dynamics change from single file to diffusive?

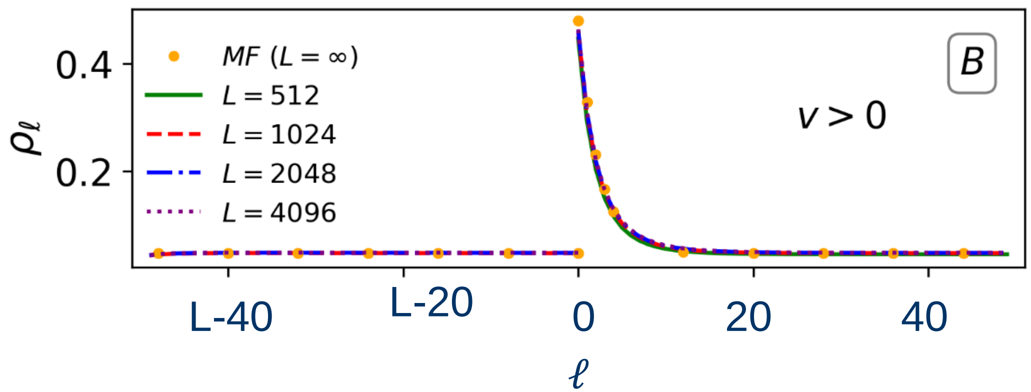
Naively one would expect a crossover to diffusive behavior at any exchange rates.

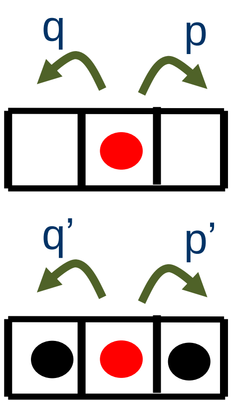
Main results

- Single file dynamics is robust to exchange (overtaking) processes.
- There is a phase transition at finite exchange rates from single file to diffusive dynamics.
- The transition is continuous.
- Multiple tracers: For a broad range of the model's parameters tracers strongly attract each other with an effective potential that grows linearly with the distance, $V(r) \propto |r|$.

Density profile at low average density

$$\bar{\rho} = 0.05$$





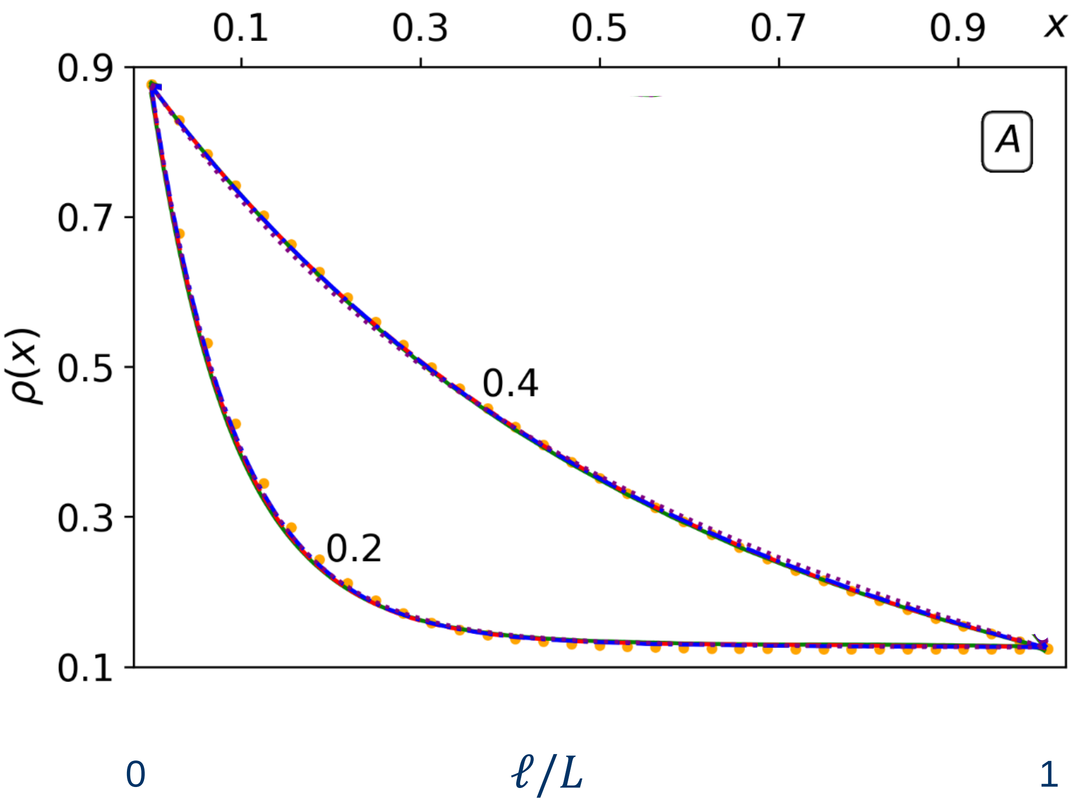
$$p = 1.75, q = 0.25, p' = 0.25, q' = 1.75$$

Finite layer of particles accumulated ahead of the tracer

The density profile is localized

Density profile in the "single file" phase

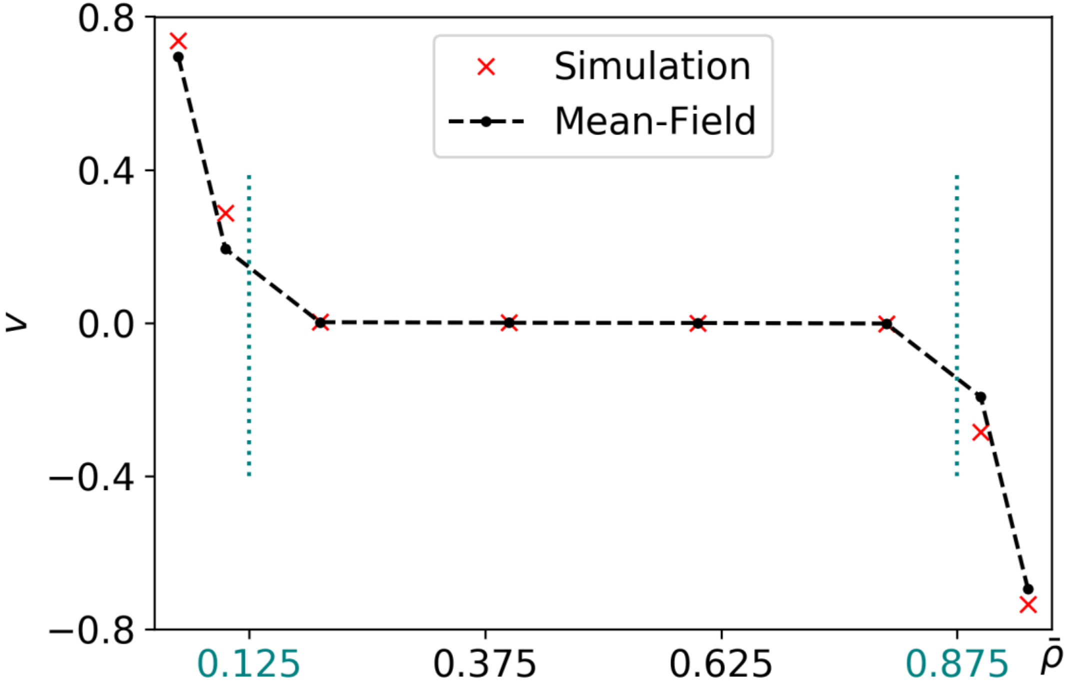
(intermediate density)



$L = 512, 1024, 2048, 4096$

The density profile extends throughout the system

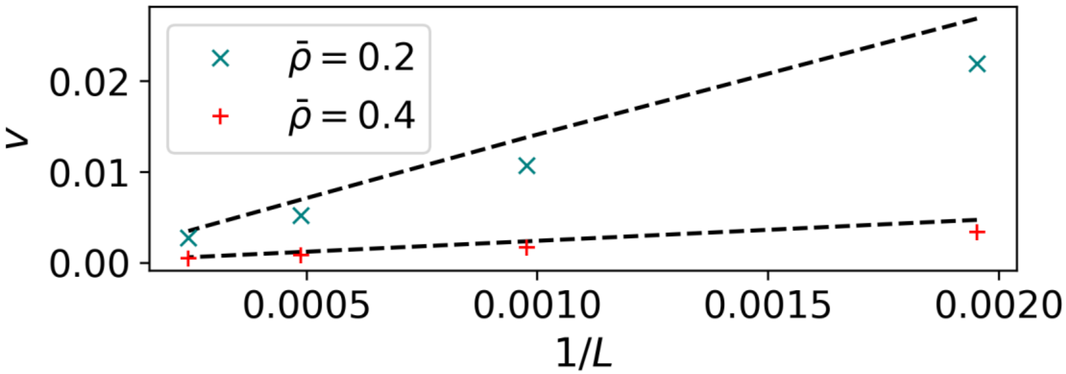
Velocity vs density



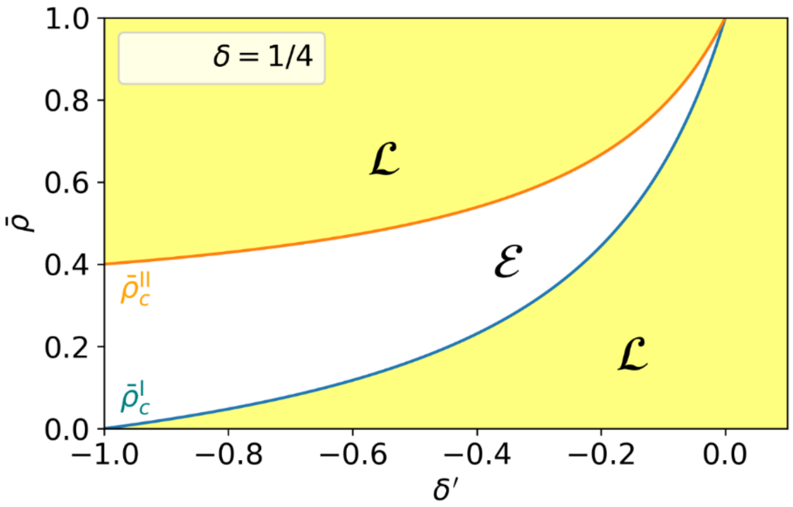
$p = 1.75, q = 0.25, p' = 0.25, q' = 1.75$

L=4096

In the single file phase



Phase diagram



$$\delta = \frac{p - q}{p + q}$$

hopping bias

$$\delta' = \frac{p' - q'}{p' + q'}$$

exchange bias

Density profile

$$\rho_l = A + Be^{-cl}$$

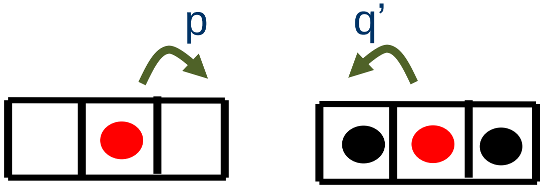
A, B, c are determined by the boundary equations and the average density

- In the single file phase $c = \frac{a}{L}$ $v = O(\frac{1}{L})$
- In the localized phase $c = O(1)$ $v = O(1)$
- At the transition $c = b/\sqrt{L}$ $v = O(1/\sqrt{L})$

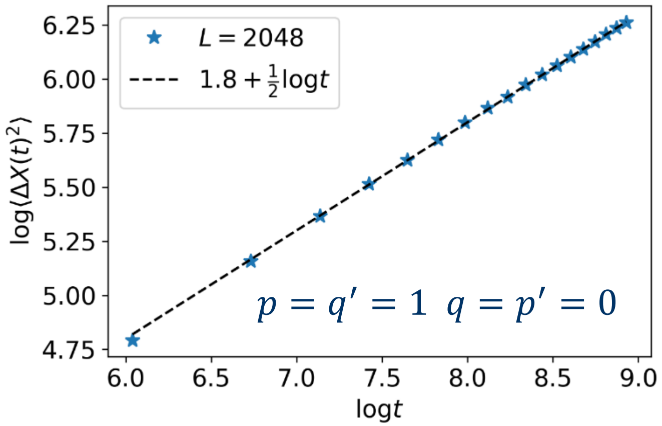
Mean-field approximation; solvable dynamical toy models

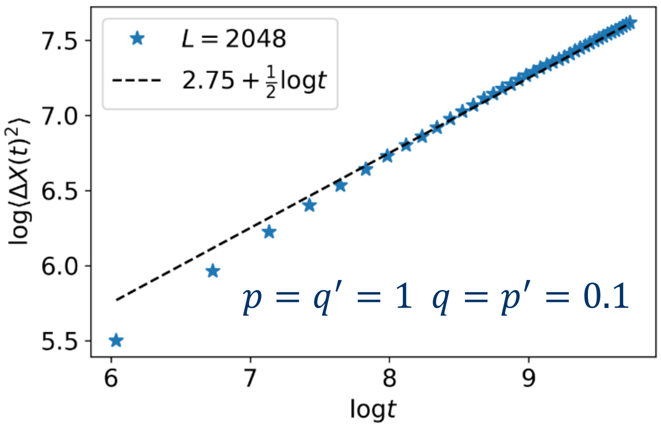
The profile coincides with the exact solution available for the case $p = q' = 1, q = p' = 0$

Derrida, Lebowitz, Speer (2002)
Blyth, Evans (2007)

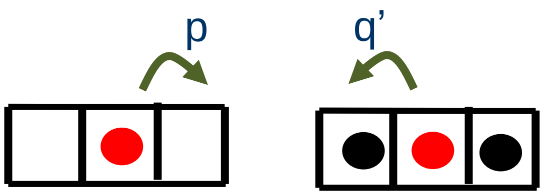


MSD of the tracer in the SF phase



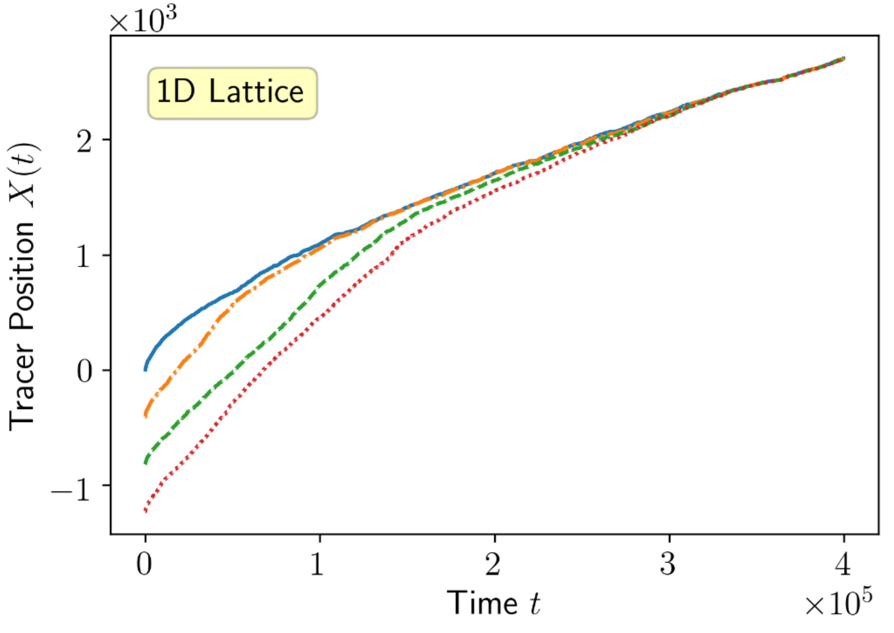


For $p = q' = 1 \; q = p' = 0$ the dynamics of the tracer is that of a biased random walker with time dependent rates decaying as $1/\sqrt{t}$



How do driven tracers interact?

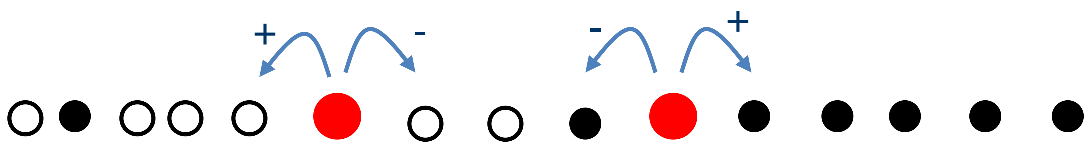
tracers tend to strongly attract each other



What is the effective potential between two tracers mediated by the bath?

Mejia-Monaserio, Oshanin (2011) – pairing of tracers in 2d
Kunster, Storm (2017) – formation of a plug in a cylinder
Poncet, Benichou, Demery, Oshanin (2019) – attraction in dense single file

Attraction mechanism



+ outward moves   - inward moves

$$\partial_t P_{n_0,n_1} = R_+^h\left(P_{n_0,-1,n_{n_1}} - P_{n_0,n_1}\right) + R_-^h\left(P_{n_0,+1,n_1} - P_{n_0 n_1}\right) + R_+^x\left(P_{n_0,n_{n_1}+1} - P_{n_0,n_1}\right) + R_-^x\left(P_{n_0,n_1+1} - P_{n_0 n_1}\right)$$

with boundary condition

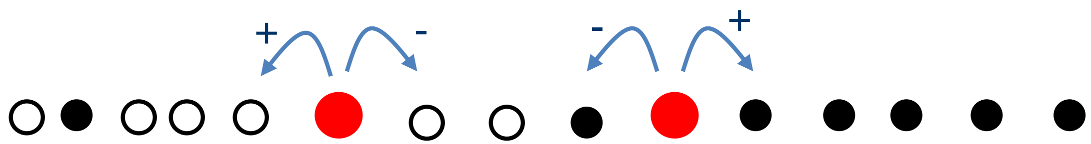
$P_{n_0,n_1} = 0$ for $n_0 \,, n_1 < 0$

$R_+^h \,, (R_-^h)$ Outward (inward) hopping rate

$R_+^x \,, (R_-^x)$ Outward (inward) exchange rate

e.g. $R_+^h = \mathrm{p}(1 - \rho_1) + q(1 - \rho_{-k-2})$

Attraction is easily understood for $p = O(1)$; $p', q', q \ll 1$; $q' \ll p'$



R_+^h, R_+^x may be estimated from the single tracer density profile

$$R_+^h, R_+^x \ll 1$$

$$R_-^h = (p+q)\frac{n_0}{n_0+n_1} \quad R_-^x = (p'+q')\frac{n_1}{n_0+n_1}$$

Outward rates << Inward rates

$$\rho_1 = \bar{\rho}\frac{p-(1-\bar{\rho})(q-q')}{p\,\bar{\rho}+p'(1-\bar{\rho})}$$

$$\rho_{-k-2} = \bar{\rho}$$

The resulting distribution (binomial):

$$P_{n_0,n_1} \propto \binom{n_0+n_1}{n_0} a^{n_0} b^{n_1}$$

$$a = \frac{R_+^h}{p+q} \qquad b = \frac{R_+^x}{p'+q'}$$

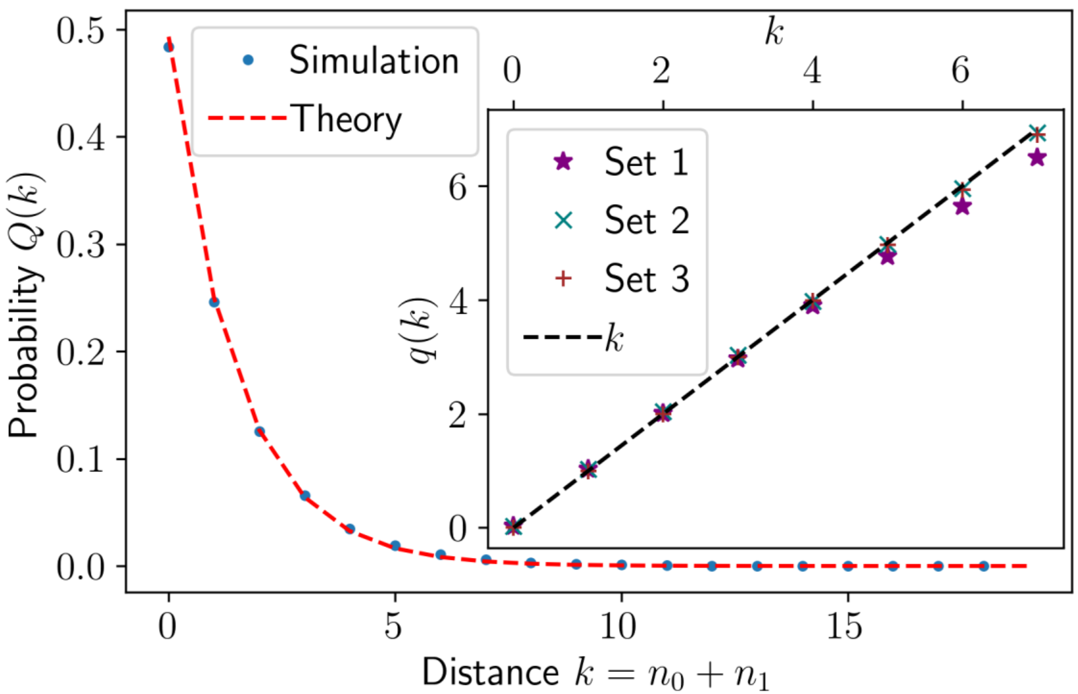
The tracers distance distribution $k = n_0 + n_1$

$$Q(k) \equiv \sum_{n_0+n_1=k} P_{n_0,n_1}$$

$$Q(k) \propto (a+b)^k$$

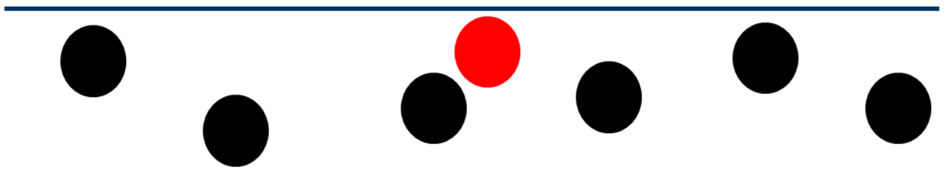
Linear effective potential

$$\beta V(k) \propto |k|$$



$$Q(k) = (1 - a - b)e^{q(k)\ln(a+b)} \qquad q(k) = k$$

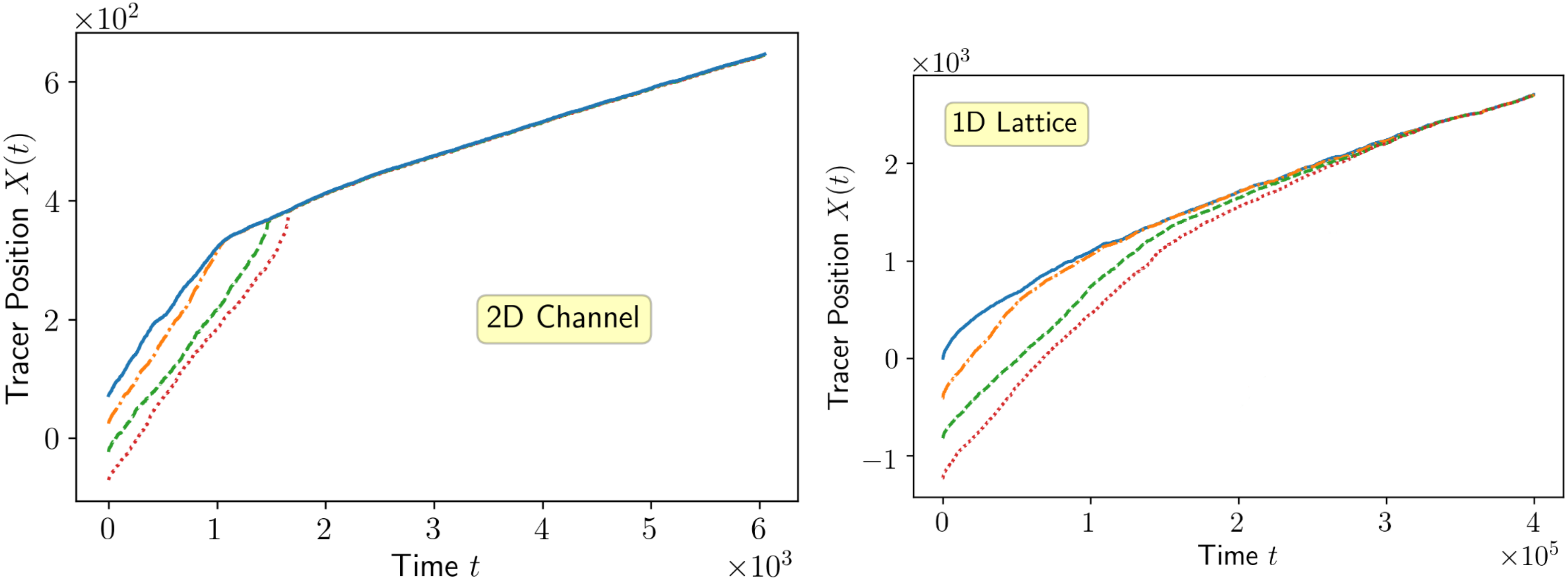
The results of the 1d model compare well with the hard disks model in a channel



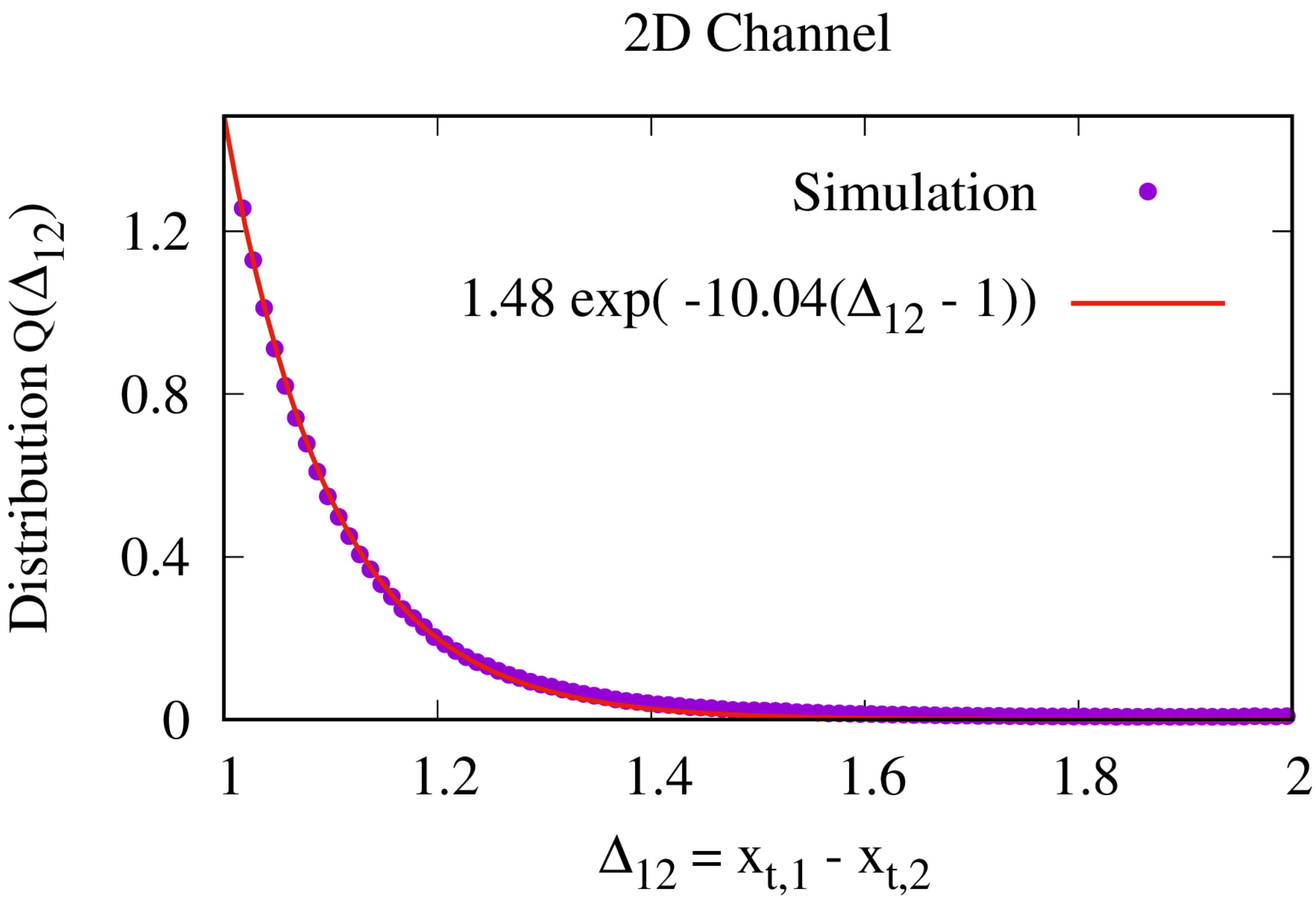
- The 2d model exhibits only the localized phase of the single tracer
- It displays strong attraction between tracers

$$\dot{\boldsymbol{r}} = \boldsymbol{v}$$

$$\dot{\boldsymbol{v}} = -\gamma \boldsymbol{v} + \sqrt{2\gamma k_B T}\boldsymbol{\xi} + (\boldsymbol{F}) + \{coll\}$$



×10^2
Tracer Position X(t)
6
4
2
0
0
1
2
3
4
5
6
Time t
×10^3
2D Channel
×10^3
1D Lattice
Tracer Position X(t)
2
1
0
−1
0
1
2
3
4
Time t
×10^5



2D Channel
Simulation
1.48 exp(-10.04(Δ_{12} - 1))
Distribution Q(Δ_{12})
1.2
0.8
0.4
0
1
1.2
1.4
1.6
1.8
2
$\Delta_{12} = x_{t,1} - x_{t,2}$

Summary

- Unlike the case of non-driven tracer: single file dynamics is robust to overtaking (particle exchange)

-

SF phase	Localized phase
$v \propto 1/L$	$v \propto O(1)$
$\langle \delta x \rangle \propto \sqrt{t}$	$\langle \delta x \rangle \propto t$
$\langle \delta x^2 \rangle \propto \sqrt{t}$	$\langle \delta x^2 \rangle \propto t$
$\rho_l = \rho^{SF}(l/L)$	$\rho_l = \rho^{B}(l)$

- Continuous transition between the two phases

- Strong attraction between tracers with $V(k) \propto |k|$ and condensation.